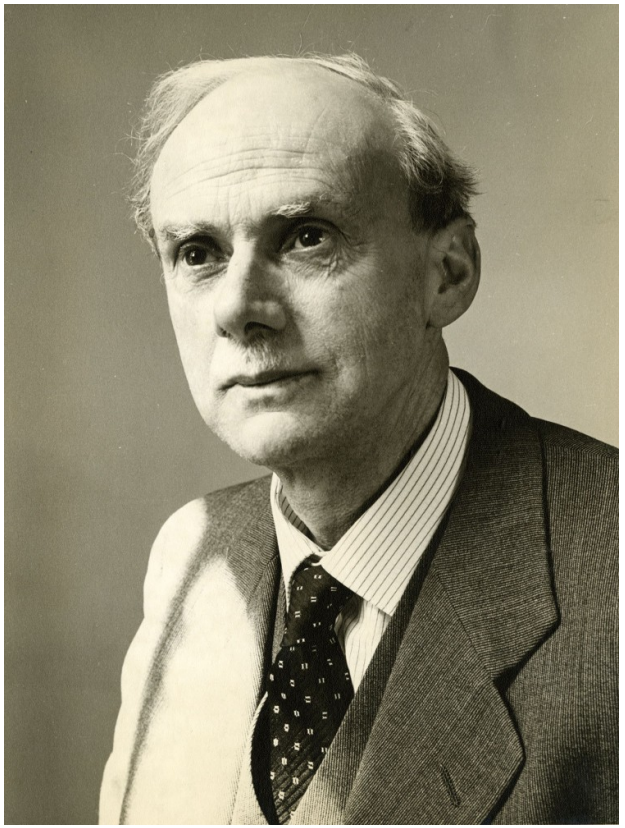


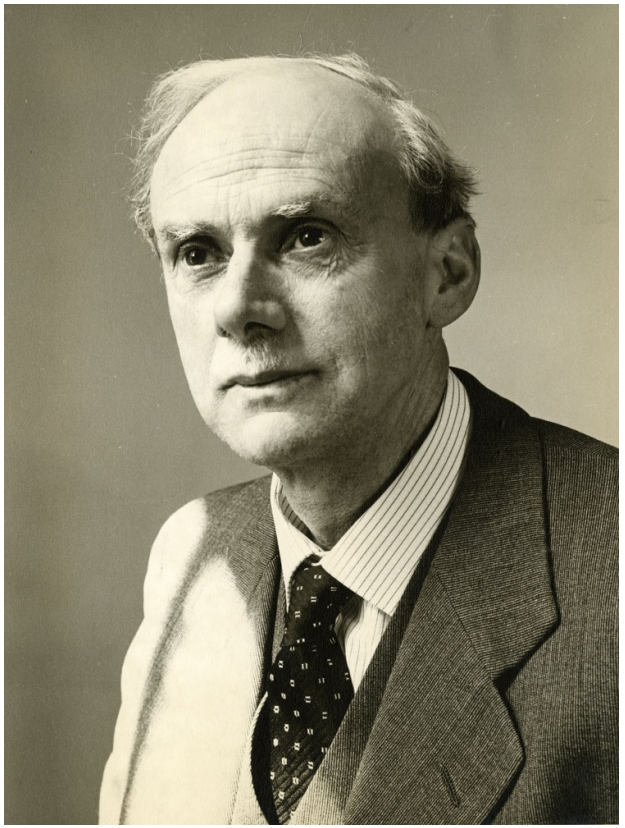
Topological matter

from Dirac, Weyl and Majorana to quantum computers

Stefanos Kourtis

PRINCETON UNIVERSITY

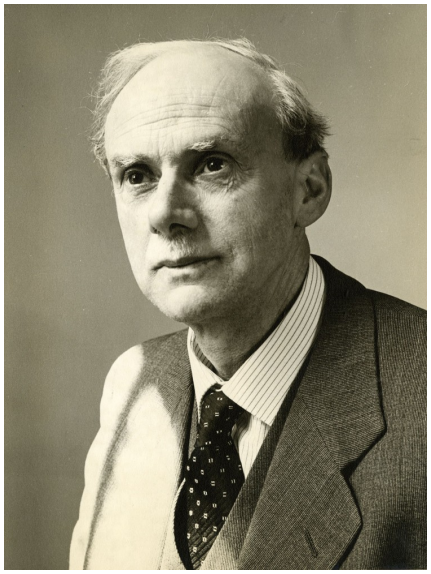
ICAM-I2CAM
Institute for Complex Adaptive Matter



"I like to play about with equations, just looking for beautiful mathematical relations which maybe don't have any physical meaning at all. ***Sometimes they do.***"
– Paul Dirac

"My work has always tried to ***unite the true with the beautiful*** *and when I had to choose one or the other, I usually chose the beautiful."*
– Hermann Weyl

"I have found a representation where all Dirac γ matrices are real. In this representation it is possible to have a real spinor that describes ***a particle identical to its antiparticle.***"
– Ettore Majorana



$$(\mathrm{i}\gamma^\mu \partial_\mu - m)\psi = 0$$

$$\gamma^0 = \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}$$

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

massless Weyl fermions: $m = 0$

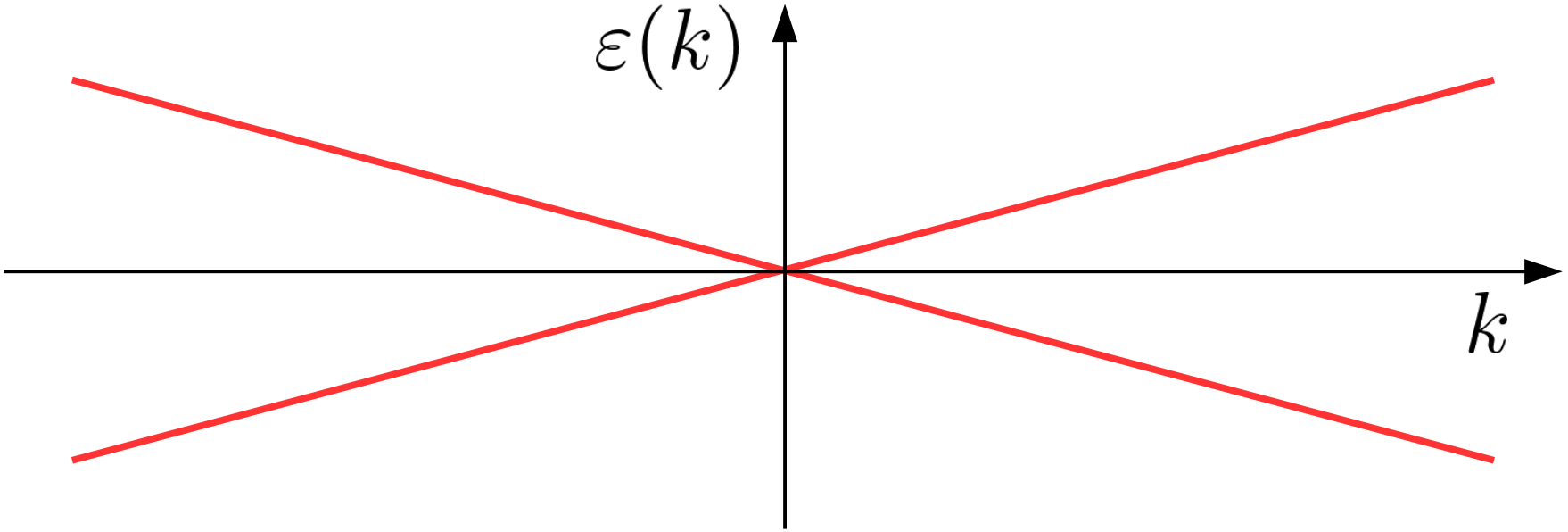
$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_1 + \psi_2 \\ \psi_1 - \psi_2 \end{pmatrix} = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$(\mathrm{i}\partial_t \pm \mathbf{p}\cdot\boldsymbol{\sigma})\psi_{L/R} = 0$$

What is a Weyl semimetal?

Weyl, 1929; Wan *et al.*, 2011

$$k \cdot \sigma$$



Why is a Weyl semimetal topological?

Weyl, 1929; Wan *et al.*, 2011

$$\mathcal{A}(\mathbf{k}) = -i \sum_{n,\text{occupied}} \langle u_{n,\mathbf{k}} | \nabla_{\mathbf{k}} | u_{n,\mathbf{k}} \rangle,$$

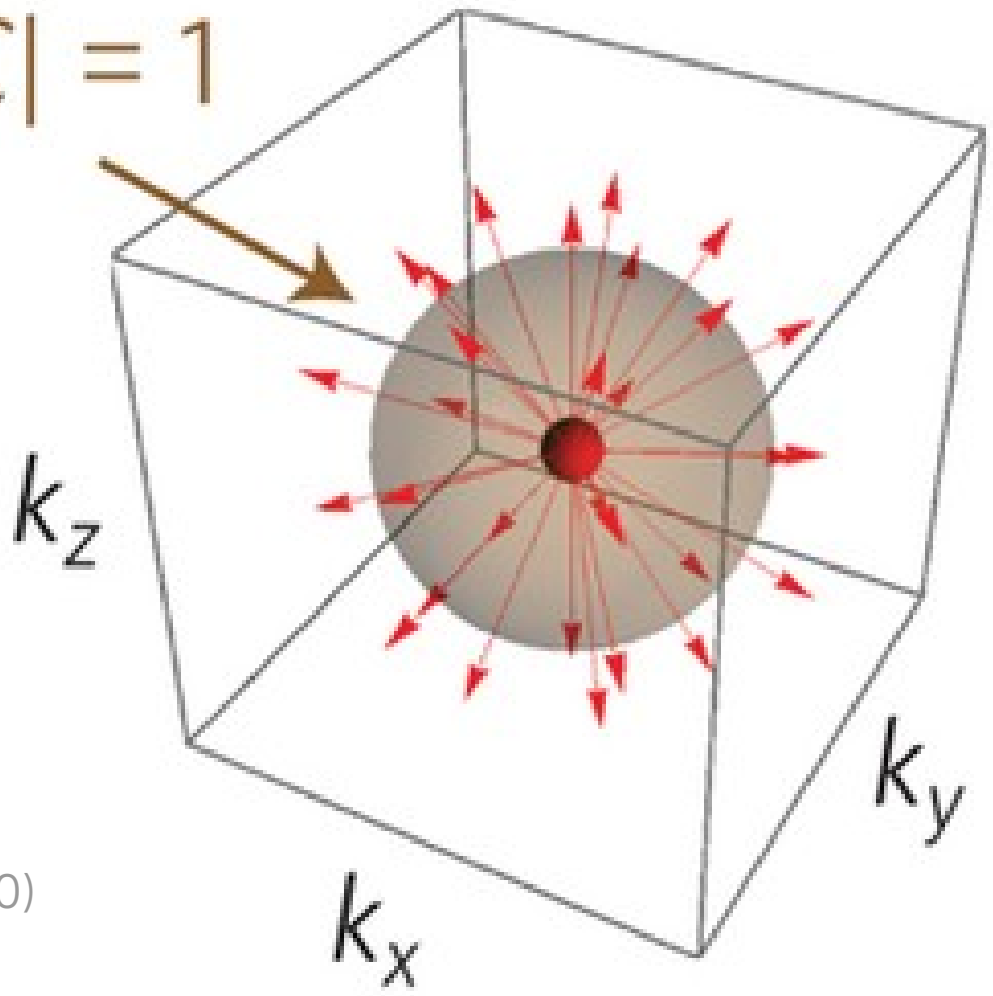
$$\mathbf{B}(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathcal{A}.$$

$$\psi(\theta,\phi) = \begin{pmatrix} \sin\frac{\theta}{2} \\ -\cos\frac{\theta}{2} e^{i\phi} \end{pmatrix}$$

$$(2\pi)^{-1} \int_S d\mathbf{k} \cdot \mathbf{B}(\mathbf{k}) = \pm 1$$

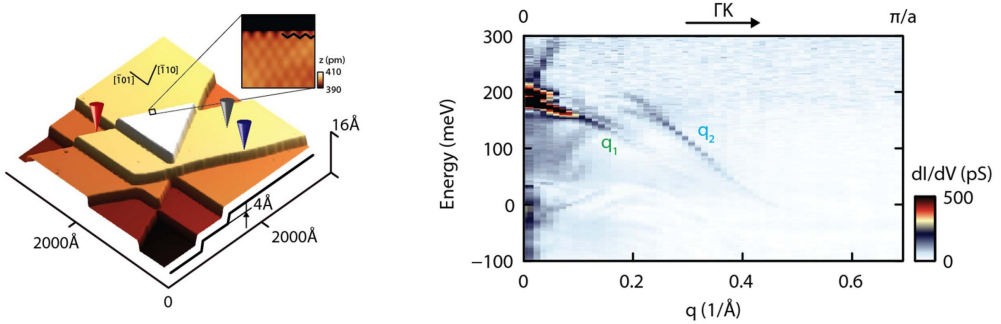
In general, the Berry curvature integrated over a closed manifold is quantized in the units of 2π and equals to the net number of monopoles inside. This number is called the Chern number and is responsible for a number of quantization effects discussed below.

Xiao, Chung, and Niu, Rev. Mod. Phys. (2010)



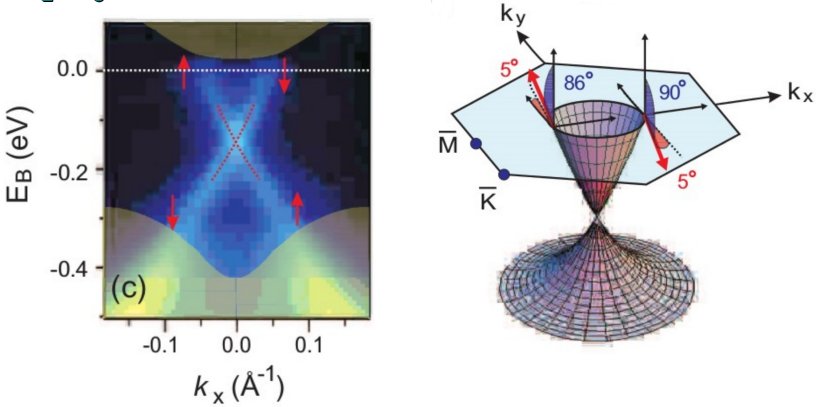
... to the laboratory

Thin film of elemental Bi: 2D topological insulator



Drozdov *et al.*, Nature Physics **10**, 664 (2014)

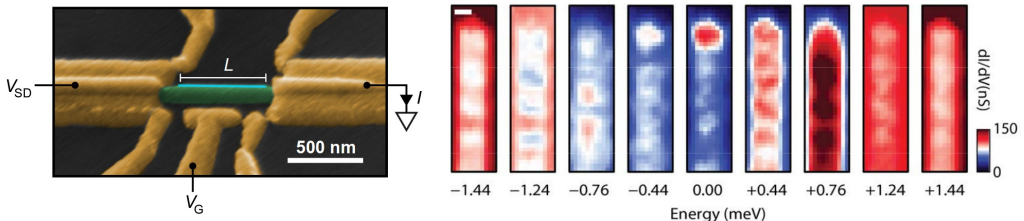
Bi_2Se_3: 3D topological insulator w/ surface Dirac fermions



Hsieh *et al.*, Nature **452**, 970 (2008)

...

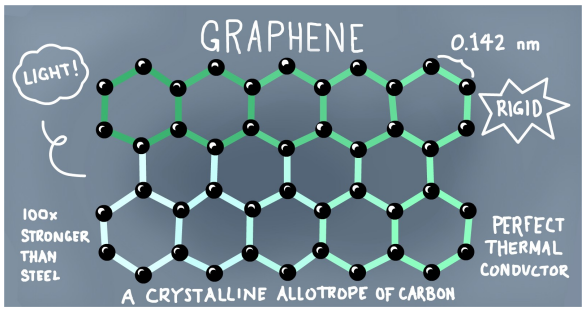
Majorana fermions in nanowires and atomic chains



Albrecht *et al.*, Nature **531**, 206 (2016)
Nadj-Perge *et al.*, Science **346**, 602 (2014)

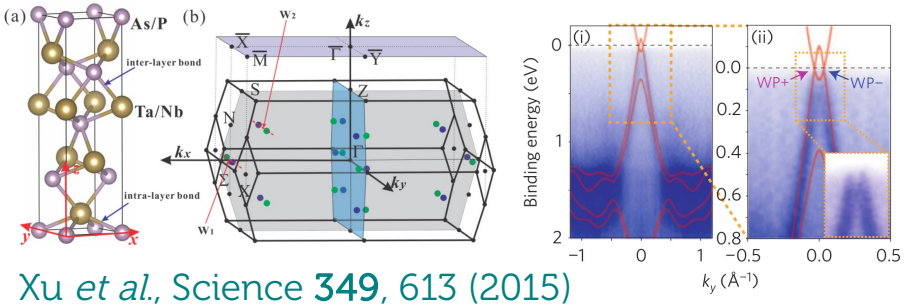
...

Graphene



Novoselov et al., Science **306**, 666 (2004)

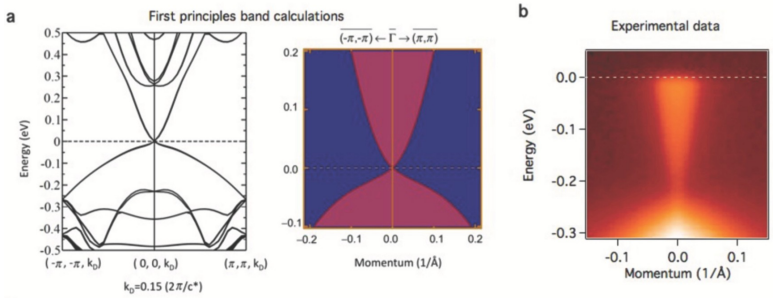
Weyl semimetals: TaAs, NbAs, TaP, NbP, ...



Xu *et al.*, Science **349**, 613 (2015)
Lv *et al.*, Nature Physics **11**, 724 (2015)

...

Dirac semimetals: Na_3Bi, Cd_2As_3, ...



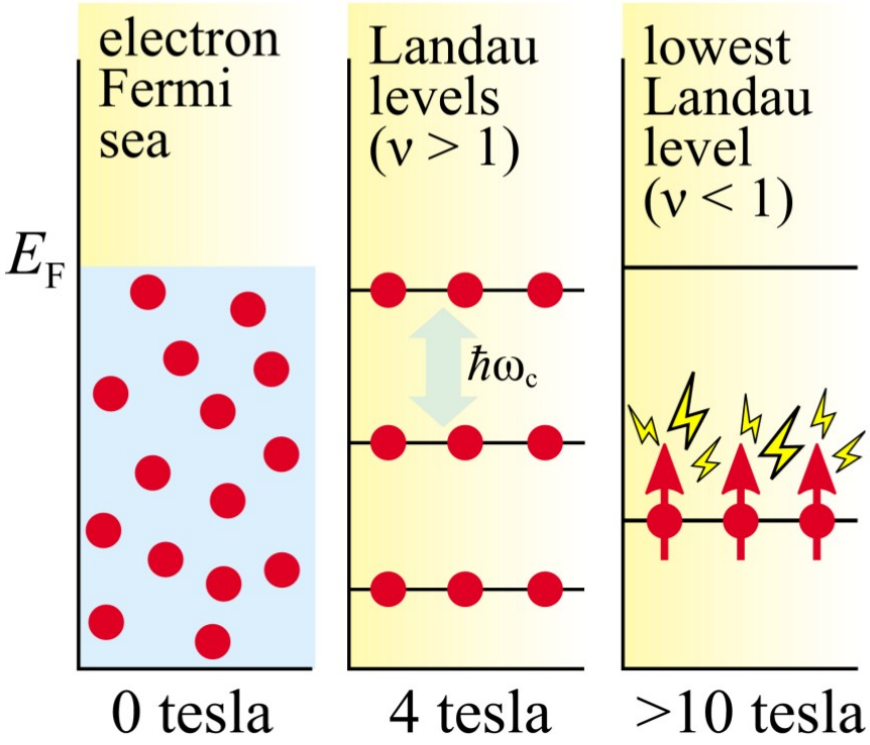
Xu *et al.*, Science **347**, 294 (2015)

...

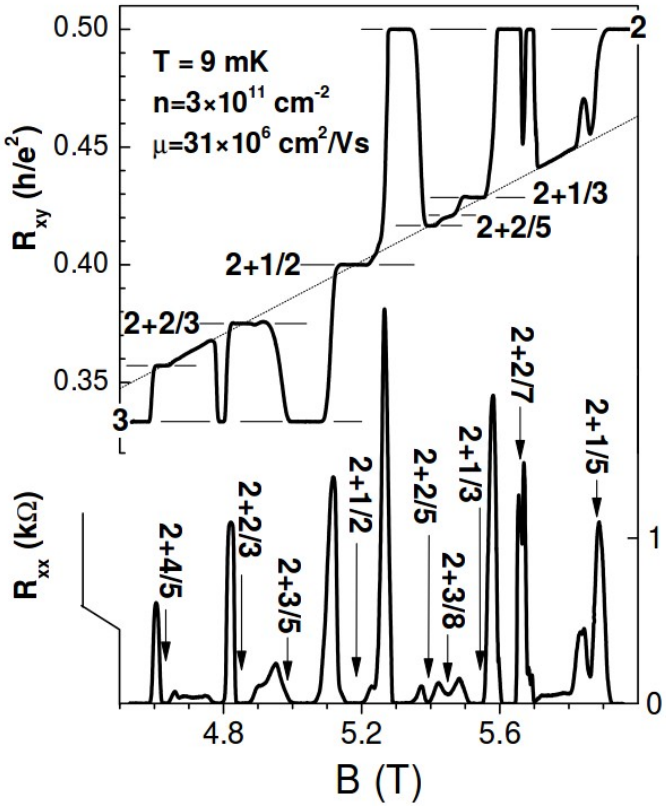
Beyond the standard model: topological order and fractionalization

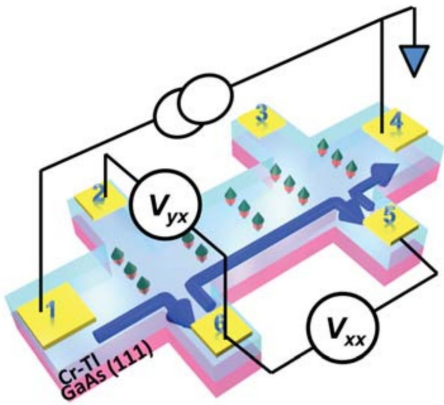
E.g.: The fractional quantum Hall effect – Tsui *et al.*, Phys. Rev. Lett. **48**, 1559 (1982)

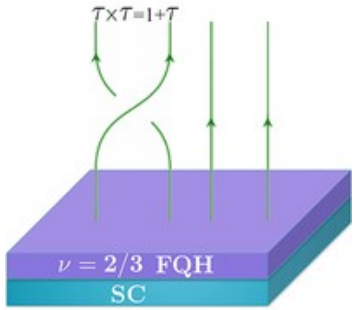
Lowest Landau level



Higher Landau levels







Braiding non-Abelian anyons → universal quantum computation

The Future™

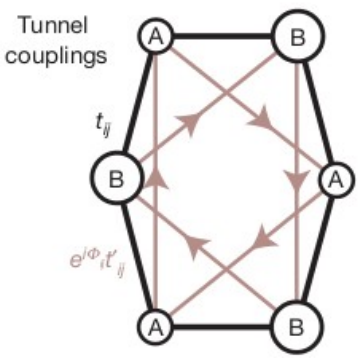
Strongly
correlated qubits
+
fractionalization
+
anyon braiding & fusion
=

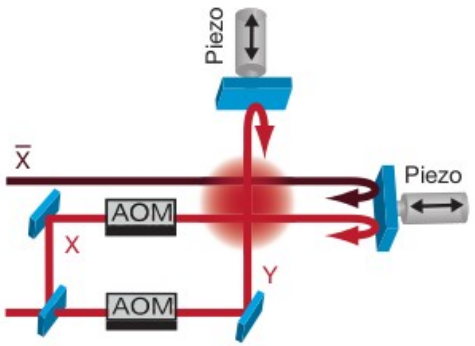
topological quantum computation

Stuff I have been working on

Topological order of cold atoms

Jotzu *et al.*, Nature (2014)
Aidelsburger *et al.*, Nat. Phys. (2015)

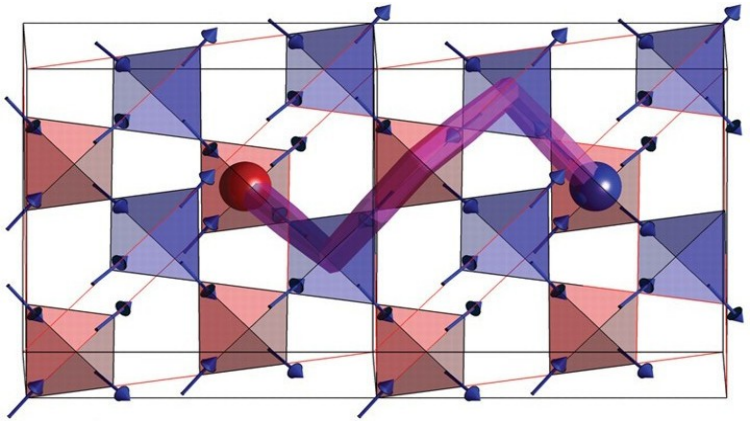




Kinetic constraints → *topological order*
→ Kourtis & Castelnovo, PRB **91**, 155134 (2015)

Dynamics in quantum spin ice

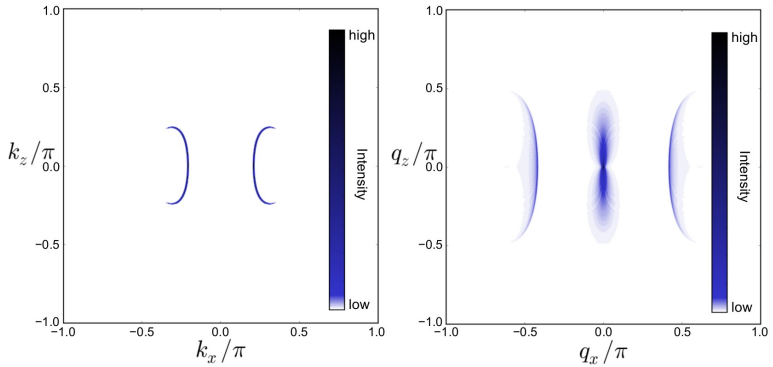
e.g., $Yb_2Ti_2O_7$ – Hallas *et al.*, PRB (2016)



Monopoles → spinons; how *free*?
→ Kourtis & Castelnovo, 1604.03951

Spectroscopy of Weyl semimetals beyond ARPES

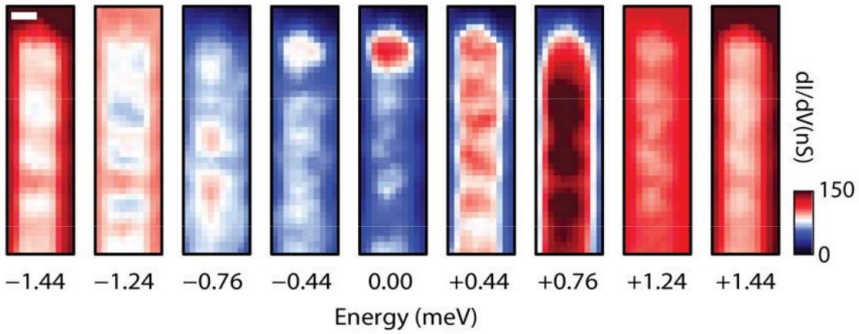
Inoue *et al.*, Science (2016)



→ Kourtis, Li, Wang, Yazdani & Bernevig,
PRB **93**, 041109(R) (2016)
→ Kourtis, 1604.03948

Probing Majorana fermions

Nadj-Perge *et al.*, Science (2014)



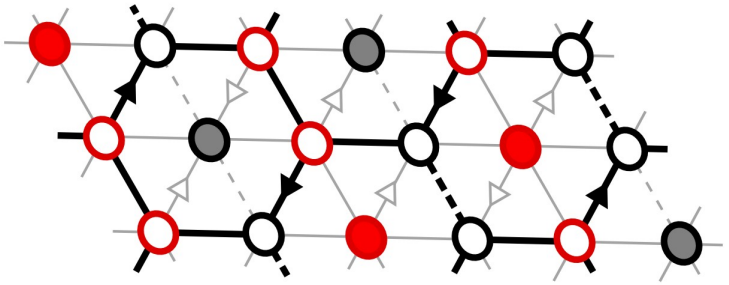
Can we see *nonlocality*? What about *correlations*?
→ Li, Kourtis & Bernevig, coming soon....

The frontier: strong correlations

Symmetry-breaking topological order

Interplay between charge, spin, orbital
(and topological?) ordering in *complex oxides*

- devise effective theories & efficient numerics for *combined* Landau & topological order
  Kotetes & Kourtis, work in progress

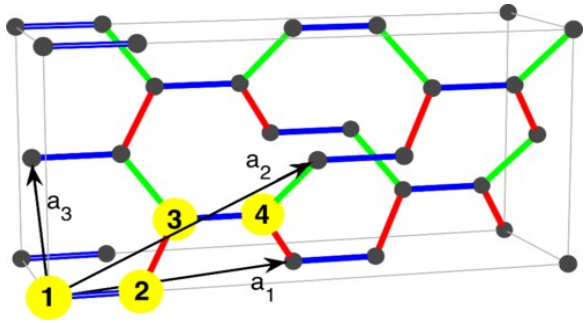


Simple example:
Kourtis & Daghofer, PRL 2014

e.g., β-Li_2IrO_3 (Smith *et al.*, 2015)
α-$RuCl_3$ (Banerjee *et al.*, 2015)

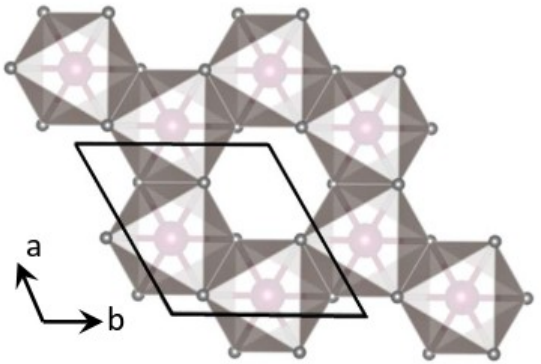
Exploit bulk topological matter

- use bulk probes → *x-rays*, neutrons, ...
- build simple *lattice* models of 3D topo order
- understand excitation *dynamics*



Realistic topologically ordered solids

- explore *strong-coupling* limit further
  preliminary work: Kourtis, Neupert, Chamon, Mudry, PRL 2014
- employ methods suitable for *correlated* topological material structures



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ΚΑΙ ΦΥΣΙΚΩΝ ΕΠΙΣΤΗΜΩΝ

ΑΡ.ΠΡΩΤ. : 6920
ΑΡ.ΔΙΠΛ. :

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ΗΜΕΡΟΜΗΝΙΑ ΠΡΩΤΗΣ ΕΓΓΡΑΦΗΣ : 19/09/2002

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ΤΑ ΑΠΟΤΕΛΕΣΜΑΤΑ ΤΩΝ ΔΙΠΛΩΜΑΤΙΚΩΝ ΕΞΕΤΑΣΕΩΝ ΕΚΔΟΘΗΚΑΝ
ΤΗΝ 05/11/2009, ΗΜΕΡΟΜΗΝΙΑ ΚΤΗΣΗΣ ΤΟΥ ΔΙΠΛΩΜΑΤΟΣ
Ο ΑΝΩΤΕΡΩ ΠΗΡΕ ΒΑΘΜΟ ΔΙΠΛΩΜΑΤΟΣ **7,24 (ΛΙΑΝ ΚΑΛΩΣ)**
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ΑΘΗΝΑ, 12-11-2009

ΜΕ ΕΝΤΟΛΗ ΤΟΥ ΠΡΥΤΑΝΗ
Η ΓΡΑΜΜΑΤΕΑΣ ΤΗΣ ΣΧΟΛΗΣ

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